(a) To find, in F_1, the curves, or segments, which remain invariant under G. This is the problem of biological stabilization, or checkmate of evolution.

(b) To study the variations, in the individual, of any curve that is checkmated in the line of heredity. For example, the nearly circular curve of blood temperature varies, during a fever, according to a cyclical case of G, viz., when, for some N, $u_1 + \cdots + u_N = 0$, $v_1 + \cdots + v_N = 0$.

(c) Analysis of the noncyclical case of G, assuming that natural selection is operative in the line of heredity. Here there is an absolute minimal (extremal), in F, and a succession of relative minima which expand as time increases. Their expansion requires that F should diverge as a system. Thus the characters of a species (and hence the typical organism itself) gradually increase in size. Lamarck formally assumed this.

LANSDOWNE, PA., U. S. A.

STRUCTURE OF RANDOM NETS

ANATOL RAPOPORT AND RAY SOLOMONOFF

Consider an aggregate of n points, from each of which issue a outwardly directed lines, each of which terminates upon another point at random. The resulting configuration constitutes a random net. Problems arising in connection with such nets have applications in mathematical theories of the central nervous system, in probabilistic theories of epidemics, and in mathematical sociology.

The existence of a "path" in a random net from a point A to another point B implies the possibility of tracing directed lines from A, through any number of intermediate points upon which they terminate, to B. One problem to be considered is the following: given a pair of arbitrarily selected points A and B, to find the expression for the probability that a path exists from A to B in terms of n and a (weak connectivity). Another problem concerns the probability that there exist paths from an arbitrarily selected point A to all the other points in the aggregate (strong connectivity). If n and a are small, the problems can be directly attacked by the method of Markoff chains. For large n, however, the amount of computation becomes prodigious and approximation methods must be used. These methods lead to either the differential-difference equation,

$$\frac{dx}{dt} = k[x(t) - x\,(t - \tau)]\,[n - x(t)],$$

or the difference equation,

$$x(t + 1) - x(t) = [n - x(t)]\,[1 - \exp\{-a[x(t) - x(t - 1)]/n\}].$$

Actually, only the asymptotic value $X = x(\infty)$ is required. For large n, this value can be approximated by the solution of the transcendental equation

$$X = n - (n - 1)\exp\{-aX/n\}.$$

If in the embryonic development of a nervous system the neuroblasts are represented by the points of our aggregate, and the axones by the directed lines, then the weak and strong connectivities measure the number of associations which may be learned by the resulting neural net.

If the points are individuals in a closed community, in which an epidemic is spreading, where a stricken individual is capable of infecting others only during a limited time and thereupon becomes non-infectious and immune (or dead), then the weak connectivity measures the expected total number of infections by the time the epidemic has passed, and strong connectivity is the probability that everyone will succumb to the disease.

Similar considerations may be applied to the spread of rumors, panics, etc.

University of Chicago,
Chicago, Ill., U. S. A.

THE USE OF THE NULL-UNIT FUNCTION IN GENERALIZED INTEGRATION

J. J. Smith and P. L. Alger

This paper develops a procedure for nth order integration, including negative and fractional, as well as integral values of n for a certain class of functions. This class of functions embraces all the usual mathematical functions, with the provision that each one is multiplied by the null-unit function $H(x, x_0)$, so that the resulting function is zero when $x < x_0$ and takes its usual value when $x > x_0$. By limiting the study to this type of function it is found that the difficulties encountered in the present theory when the null-unit function is not used are avoided. The development proceeds in three steps.

A definition of generalized integration proposed by Liouville is taken as the starting point, since this reduces to the standard definition for an integral when n is 1 and to the definition for a differential coefficient when n is -1. It is found that, while this definition is satisfactory for a function such as e^x, it gives indeterminate results for functions such as x^m.

The second step is to modify the definition by limiting the integration to a given interval, from x_0 to x. This overcomes the difficulty in integrating x^m. However, it is found that nth order differentiation so defined (n negative) is not the inverse of a succeeding nth order integration (n positive). With this non-commutative process, the so-called constants of integration must be found, offering serious limitations to the solution of practical problems.

The third step, leaving the definition unchanged, is to introduce the null-unit function, $H(x, x_0)$ as a factor, so that the integrated function is equal to zero when $x < x_0$. It is then found that the processes of integration and differentiation become commutative, for all values of n, in the same way as shown in a previous paper for integral values of n.